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SUPERSYMMETRY AND GRAND UNIFICATION

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Abstract

Supersymmetry and Grand Unification are the two most promising directions for physics beyond the Standard Model. They receive indirect experimental support from the apparent lightness of the Higgs boson, the values of the gauge couplings measured at LEP and elsewhere, and the persistent solar neutrino deficit. Phenomenological constraints and theoretical models constrain predictions in interesting ways. All these ideas may be embedded in string theory, which is shown by newly-discovered dualities to possess previously-unsuspected richness and simplicity.

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Beyond the Standard Model

Although the Standard Model (SM) is in perfect agreement with (almost) all experimental data, theorists are not content with it and believe that something must lie beyond it. It is common to categorize the open problems left by the SM into the **problem of unification**, which motivates the search for a simple gauge theory that contains all the gauge forces, **the problem of flavour**, namely why are there so many different types of quarks and leptons, and what explains their weak mixing and CP violation, and the **problem of mass**. This includes not only the question of the origin of the particle masses, to which the SM answer is an elementary Higgs boson, but also why all the SM particle masses are so small, to which one possible answer may be provided by supersymmetry, as we shall discuss in the rest of this talk. All these problems should be resolved in a Theory Of Everything (TOE) which includes gravity and reconciles it with quantum mechanics. Such a theory should also explain the origin of spacetime, why we live in four dimensions and many other fundamental problems of particle physics and cosmology. The only candidate we have for such a TOE is the superstring, which will also be discussed at the end of this talk.

1. Motivations for Supersymmetry

Supersymmetry[1] is a beautiful theory, but the motivations for it to appear at accessible energies are related to the problem of mass mentioned above, namely the origin of the hierarchy of mass scales in physics, and its naturalness in the presence of radiative corrections[2]. The question why m_W is much less than $m_{\rm Planck}$ or $m_{\rm GUT}$ can be rephrased as a question: Why is $G_F \gg G_N$, or even why the Coulomb potential inside an atom is much stronger than the Newtonian potential:

$$\frac{e^2}{r} \lesssim G_N \times \frac{m^2}{r} \tag{1}$$

This hierarchy is valuable to radiative corrections. We say that a theory is natural if the radiative corrections are not much larger than the physical values of observable quantities. For example, the leading one-loop correction to a fermion mass takes the form

$$\delta m_f = 0\left(\frac{\alpha}{\pi}\right)\ m_f\ \ln\left(\frac{\Lambda}{m_f}\right) \tag{2}$$

which is not much larger than m_f for any reasonable cut-off $\Lambda \lesssim m_P$.

Naturalness is, however, a problem for an elementary Higgs boson, which in the electroweak sector of the SM must have a mass

$$m_H = 0\left(\sqrt{\frac{\alpha}{\pi}}\right)^{0\pm1} \times m_W \tag{3}$$

The one-loop diagrams shown in Fig. 1 lead to "large" radiative corrections of the form

$$\delta m_H^2 \simeq g^2_{f,W,H} \int^{\Lambda} \frac{d^4k}{(2\pi)^4}\ \frac{1}{k^2} = 0\left(\frac{\alpha}{\pi}\right)\Lambda^2 \tag{4}$$

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Fig. 1. Quadratically-divergent one-loop diagrams contributing to m_H^2, m_W^2.

These are much larger than the physical value m_H^2 if the cut-off Λ, representing the scale at which new physics appears, is of order m_P or $m_{\rm GUT}$.

Supersymmetry solves the naturalness problem of an elementary Higgs boson[2] by virtue of the fact that it has no quadratic divergences and fewer logarithmic divergences[3] than non-supersymmetric theories. The fermion and boson diagrams shown in Fig. 1 have opposite signs, so that their net result is

$$\delta m_{W,H}^2 \simeq -\left(\frac{g_F^2}{4\pi^2}\right) \quad (\Lambda^2 + m_F^2) + \left(\frac{g_B^2}{4\pi^2}\right) \quad (\Lambda^2 + m_B^2)\ . \tag{5}$$

The leading divergences cancel if there are the same numbers of bosons and fermions, and if they have the same couplings $g_F = g_B$, as in a supersymmetric theory. The residual contribution is small if supersymmetry is approximately valid, i.e., if $m_B \simeq m_F$:

$$\delta m_{W,H}^2 \simeq 0\left(\frac{\alpha}{\pi}\right) \quad \left|m_B^2 - m_F^2\right| \tag{6}$$

which is no larger than $m_{W,H}^2$ if

$$\left|m_B^2 - m_F^2\right| \lesssim 1\ \mathrm{TeV}^2 \tag{7}$$

This property provides the first motivation for supersymmetry at low energies. However, it must be emphasized that this is a qualitative argument which should be regarded as a matter of taste. After all, mathematically an unnatural theory is still renormalizable, even if it requires fine tuning of parameters to obtain the correct physical values. A second supersymmetric miracle is the absence of many logarithmic divergences: for many Yukawa couplings and quartic terms in the effective potential[3],

$$\delta\lambda \propto \lambda \tag{8}$$

which vanishes if the rare coupling $\lambda = 0$. This means that couplings between light and heavy Higgses, which could devastate the hierarchy[4], will not appear via quantum corrections if they are absent at the tree level. The combination of Eqs. (5) and (8) means that if $m_W \leq m_P$ at the tree level, it stays small in all orders of perturbation theory, solving the naturalness problem and providing a context for attacking the hierarchy problem.

obtained to weigh soft

The minimal supersymmetric extension of the Standard Model (MSSM)[5] is characterized by gauge interactions which are the same as those in the Standard Model (SM), and Yukawa interactions obtained from a cubic superpotential which is an analytic function of the left-handed fields

$$W = \sum_{L,E^c} \lambda_L \; LE^c H_1 + \sum_{Q,U^c} \lambda_U \; QU^c H_2 + \sum_{Q,D^c} \lambda_D \; QD^c H_1 + \mu H_1 H_2 \tag{9}$$

The first three terms give masses to the charged leptons, charge-2/3 quarks and charge-1/3 quarks respectively. Two Higgs doublets are needed in order to preserve the analyticity of W and to cancel triangle anomalies. This implies the introduction of the fourth term in Eq. (9), which couples the Higgs supermultiplets. The quartic part of the effective scalar potential is determined by the gauge and Yukawa interactions, which leads to the relations between the physical Higgs boson masses to be discussed later.

In addition to the above supersymmetric parts of the effective action, supersymmetry breaking is necessary to obtain $m_F^2 \neq m_B^2$, which is usually parametrized by soft mass parameters for scalars m_{0_i} and gauginos $M_{1/2_a}$, as well as soft trilinear and bilinear coefficients A_{ijk} and B_{ij}. In much the same way as gauge couplings in conventional GUTs, these are subject to renormalization:

$$M_{1/2_a} \propto \alpha_a \; , \qquad \tilde{M}_{0_i}^2 - M_{0_i}^2 + C_{ia} M_{1/2_a}^2 + D_i M_Z^2 \tag{10}$$

where the coefficients C_{i_a} and D_i are calculable[6]. It is often assumed that the soft supersymmetry breaking parameters are universal at some high renormalization scale $Q = M_{GUT}$ or M_P:

$$M_{1/2_a}|_Q = M_{1/2} \; , \quad M_{0_i}^2 = M_0^2 \tag{11}$$

This assumption protects the low-energy theory against flavour-changing neutral currents (FCNC)[7], but it is not necessarily true. For example, there could be non-trivial renormalization at scales $M_{GUT} \lesssim Q \lesssim M_P$, so that:

$$(M_0^2)_{\overline{\mathbf{5}}} \neq (M_0^2)_{\mathbf{10}} \tag{12}$$

in a context of an $SU(5)$ GUT[8], and/or differences may emerge when the GUT degrees of freedom are integrated out, and/or the input parameters may not be universal at $Q = M_P$ [9]:

$$M_{O_i}^2 = f_i \; (\text{moduli}) \tag{13}$$

where "moduli" is a fancy term for vacuum expectation values in a string theory. Some such violations of universality may be consistent with the FCNC constraints[10], particularly for heavier generations.

If one nevertheless assumes universality, different experimental constraints can be combined to compile the physics reach, both present and future, as in the $(m_0, M_{1/2})$ plane shown in Fig. 2, or the $(\mu, M_{1/2})$ plane shown in Fig. 3 [11]. In each case, the diagonally-shaded regions are those excluded by present experimental constraints. Also shown in Fig. 2 are regions excluded by theoretical considerations. Both figures show the mass contours for sparticles that could be studied with future accelerators such as LEP2 or the LHC.

Fig. 2. Present experimental (shaded) and theoretical (bricked) constraints in the $(m_0, m_{1/2})$ plane, assuming universal supersymmetry breaking[11].

In addition to the search for supersymmetric particles, a promising avenue for probing supersymmetry is the search for supersymmetric Higgs bosons. The two complex Higgs doublets required in the MSSM contain eight real degrees of freedom, of which three are eaten by the $W^\pm$ and the Z^0 to give them their masses, leaving five physical Higgs bosons to be discovered. Three of these (h, H, A) are neutral and two $(H^\pm)$ are charged. At the tree level, all their masses and couplings are specified in terms of two parameters, which may be taken as m_A or m_h and $\tan\beta \equiv v_2/v_1$. These restrictions follow from the supersymmetric form of the Higgs potential, and would imply that $m_h < m_Z$ at the tree level but there are important radiative corrections[12]

which depend strongly on the mass of the top quark, which is now known[13] to be large:

$$\delta m_h^2 \simeq \frac{3g^2}{8\pi^2} \; \frac{m_t^4}{m_W^2} \; \ln\left(\frac{m_{\tilde{q}}^2}{m_t^2}\right) \tag{14}$$

Fig. 3. Present experimental constraints and future LEP2 physics reach in the $(\mu, M_2 \equiv m_{1/2}\alpha_2/\alpha_{\rm GUT})$ plane[11].

These raise the upper bound on m_h to as large as 130 GeV, as seen in Fig. 4 [14].

Before the inclusion of these radiative corrections, experimentalists at LEP2 could have been quite sure of finding the lightest neutral supersymmetric Higgs h. Even with these radiative corrections included, they are still able to explore a large fraction of the parameter space, as seen in Fig. 5 [11]. We see here the importance of increasing the centre-of-mass energy of LEP2 as high as possible. The search for supersymmetric Higgs bosons at the LHC has also been studied intensively during the past year, and Fig. 6 exhibits the domains of parameter space that may be explored by the ATLAS and CMS detectors using various supersymmetric Higgs signatures[15]. We see from Figs. 5 and 6 that LEP2 and the LHC between them should be able to explore all of the MSSM parameter space, at least if the LEP2 energy reaches 192 GeV as is now being proposed.

2. Possible Experimental Motivations for Supersymmetry

The precision electroweak data from LEP and elsewhere provide two (or three?) tentative indications favouring a supersymmetric world view. One is that they favour a relatively light

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Fig. 4. Upper limit on m_h in the MSSM as a function of $\tan\beta$ for zero (dashed) and maximal (solid) mixing, assuming $m_{\tilde{q}}$ =1 TeV [11].

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Fig. 5. Reach for Higgs bosons in the MSSM at LEP2 with a centre-of-mass energy of 192 GeV. The dark shaded regions are excluded theoretically[11].

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Fig. 6. Reach for Higgs bosons in the MSSM at the LHC[15].

Fig. 7. The values of χ^2 as a function of M_H from a global fit[16] to the precision electroweak data.

Higgs boson[16,17]. For several years, global fits have consistently given preferred values $m_H \lesssim 300$ GeV, and are highly consistent with the prediction of the MSSM that $m_h \simeq m_Z \pm 40$ GeV [12,14]. Figure 7 shows the χ^2 function for a recent global fit in the SM, which yields[16]

$$M_H = 76^{+152}_{-50} \text{ GeV} \tag{15}$$

The χ^2 obtainable in the MSSM is essentially identical[18], whilst strongly-interacting Higgs models such as those based on technicolour have much larger χ^2 and are disfavoured[19].

The second indication favouring supersymmetry is that measurements of the SM gauge couplings $\alpha_{1,2,3}$ have for some time[20,21] favoured supersymmetric GUTs over the minimal non-supersymmetric GUT, which predicts[22]:

$$\sin^2\theta_W(m_Z)\Big|_{\overline{MS}} = 0.208 + 0.004(N_H - 1) + 0.006 \ln\left(\frac{400 \text{ MeV}}{\Lambda_{\overline{MS}}(N_f = 4)}\right)$$

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Fig. 8. Gee-whizz plot showing how well GUT predictions of $\sin^2\theta_W$ agree with the experimental data.

$$= \quad 0.214 \pm 0.004 \tag{16}$$

This tendency has been strongly reinforced by the higher-precision data recently provided by LEP [23]. Figure 8 gives an overview of the present theoretical and experimental situation. The qualitative success of GUTs in predicting $\sin^2\theta_W$ is impressive: it is only when we blow the vertical scale up by a factor of 10 that we notice a discrepancy with the minimal non-supersymmetric GUT prediction in Eq. (16), and only when we blow it up by a further factor of 10 that we begin to wonder whether the LEP data on $\sin^2\theta_W$ may fall below the prediction of a minimal supersymmetric GUT. However, it should be emphasized that supersymmetric GUTs contain many parameters, reducing the precision of their predictions[24,25]: we shall return to them later.

Another experimental effect which has excited much interest recently, including speculations about supersymmetry, is the possible discrepancy between LEP measurements and the SM predictions for the rates for Z^0 decays into bottom and charm quarks R_b and R_c [23]. Some authors have investigated whether this possible discrepancy could be accommodated within the MSSM, if either supersymmetric Higgs bosons or stops and charginos are light[26], just above the mass ranges excluded by direct searches. It is possible to explain R_b, but it is very difficult to explain the central experimental value of R_c, whose numerical discrepancy with the SM value is even larger, though a smaller number of standard deviations[23]. Many theoretical models share this lack of success in explaining simultaneously R_b and R_c, and the latter would be very surprising if it were to be confirmed. My present attitude is to wait and see how these experimental discrepancies develop[27], and not yet to interpret them as evidence for supersymmetry. With the resolution of the hierarchy problem, the indication of a light Higgs boson and the GUT unification of the gauge couplings, we may already have enough motivation for supersymmetry!

3. Grand Unified Theories

Now is the time to delve deeper in the guts of GUTs, reviewing the extent to which they accentuate the hierarchy problem, studying in more detail the correlation they provide between the values of $\alpha_s(M_Z)$ and $\sin^2\theta_W$, and reviewing their predictions for novel phenomena such as

proton decay and neutrino masses.

The hierarchy problem reviewed in Section 2 can be restated as the question: "Why is the electroweak Higgs boson light?" In the context of the minimal $SU(5)$ GUT, this question can be reformulated as: "Why is $m_{\mathbf{2}} \leq m_{\mathbf{3}}$?", where the subscripts denote the doublet and triplet components of the five-dimensional Higgs representations. The enormous separation between these masses is done by hand in the minimal $SU(5)$ model[28]:

$$\left.\begin{array}{l} m_{\mathbf{3}} \\ m_{\mathbf{2}} \end{array}\right\} = m_{\mathbf{5}} \left\{ \begin{array}{c} +2 \\ -3 \end{array} \right\} \lambda < 0|V_{\mathbf{24}}|0 >= \left\{ \begin{array}{l} 0(M_{\mathrm{GUT}}) \\ 0(M_W) \end{array} \right. \tag{17}$$

which requires inelegant and extreme fine-tuning between the bare and $< \mathbf{24} >$ contributions $m_{\mathbf{5}}$ and $-2\lambda < 0|V_{\mathbf{24}}|0 >$ to the doublet mass $m_{\mathbf{2}}$. An improvement is possible in principle in missing-partner models[29], in which the triplet Higgs components require large Dirac masses from couplings with other triplet fields, but there are no such partners for the doublet fields, which therefore remain light. This is an elegant idea, but its realization in conventional GUTs is very complicated, requiring several large Higgs representations, such as[30]

$$\begin{array}{rl} SU(5): & \mathbf{50} + \overline{\mathbf{50}} + \mathbf{75} + \ldots \\ SO(10): & 3.\mathbf{16} + 2.\mathbf{10} + 3.\mathbf{45} + \mathbf{54} + \overline{\mathbf{126}} + \mathbf{126} \end{array} \tag{18}$$

The simplest missing-partner mechanism is that [31] in the flipped $SU(5) \times U(1)$ GUT[32], in which the GUT Higgses occupy $\mathbf{10}$ and $\overline{\mathbf{10}}$ representations, and the triplet components of the five-dimensional electroweak Higgs representations couple to triplet components of the GUT Higgses to require large Dirac masses. Examples of flipped $SU(5) \times U(1)$ GUTs have been derived in string theory[33]. However, the other potential solutions to the hierarchy problem are problematic in string models: in general, these do not allow bilinear mass terms of the type required in Eq. (17), exotic representations like those in Eq. (18) are not found[34], and their pattern of couplings may also be difficult to arrange.

We now explore in more detail the supersymmetric GUT relation between $\alpha_s(M_Z)$ and $\sin^2\theta_W$ [35]. When one looks more carefully at the gee-whizz plots of the gauge couplings in the MSSM meeting at a single Grand Unification scale around 10^{16} GeV, one finds a possible minor discrepancy with the minimal supersymmetric GUT as already mentioned in the context of Fig. 8. The supersymmetric GUT prediction for $\sin^2\theta_W$ can be written in the form[24]

$$\begin{aligned} \sin^2\theta_W(M_Z)\Big|_{\overline{MS}} \;=\; & 0.2029 + \frac{7\alpha_{em}}{15\alpha_3} + \frac{\alpha_{em}}{20\pi}\Big[-3\ln\left(\frac{m_t}{M_Z}\right) + \frac{28}{3}\ln\left(\frac{m_{\tilde{g}}}{M_Z}\right) \\ & -\frac{32}{3}\ln\left(\frac{m_{\tilde{W}}}{M_Z}\right) - \ln\left(\frac{M_A}{M_Z}\right) - 4\ln\left(\frac{\mu}{M_Z}\right) + \ldots \Big] \end{aligned} \tag{19}$$

which involves many supersymmetry-breaking parameters. It is convenient to summarize these in the lumped parameter[36]

$$T_{\mathrm{SUSY}} \equiv |\mu| \left(\frac{m^2_{\tilde{W}}}{m^2_{\tilde{g}}}\right)^{14/19} \left(\frac{M^2_A}{\mu^2}\right)^{3/38} \left(\frac{m^2_{\tilde{W}}}{\mu^2}\right)^{2/19} \prod_{i=1}^{3} \left(\frac{m^3_{\tilde{l}_i} m^7_{\tilde{q}_i}}{m^2_{\tilde{e}_i} m^5_{\tilde{u}_i} m^3_{\tilde{d}_i}}\right)^{1/19} \tag{20}$$

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Fig. 9. Minimal supersymmetric $SU(5)$ GUT predictions for $\alpha_s(M_Z)$ [39].

If one further assumes universality at the Grand Unification scale, then approximately

$$T_{\rm SUSY} \simeq \mu \left(\frac{\alpha_2}{\alpha_3}\right)^{3/2} \simeq \frac{\mu}{7} \tag{21}$$

It should be noted that $T_{\rm SUSY} \sim 300$ GeV corresponds to squark masses around 2 TeV. The prediction (19) is to be compared with the experimental value[37]

$$\begin{aligned} \sin^2\theta_W(M_Z)_{\overline{MS}} &= 0.2317 \pm 0.0003 + (5.4\times 10^{-6}) \quad (m_H - 100 \text{ GeV}) + \ldots \\ & \quad -(3.03\times 10^{-5})\ (m_t - 165 \text{ GeV}) + \ldots \\ \\ &= 0.2312 \pm 0.0003 \quad \text{for} \quad m_H = 100 \text{ GeV}, \quad m_t = 180 \text{ GeV} \end{aligned} \tag{22}$$

where the effects of M_H and m_t have been indicated explicitly, but there are additional supersymmetric corrections[38] which may reach the per cent level. This comparison yields[39]

$$\begin{array}{lll} \alpha_s(M_Z) > 0.126 & \text{for} & T_{\rm SUSY} < M_Z \\ \text{or} \qquad\quad > 0.121 & \text{for} & T_{\rm SUSY} < 300 \text{ GeV} \end{array} \tag{23}$$

as seen in Fig. 9, with an error of about 0.0015.

Before concluding that supersymmetric GUTs favour values of $\alpha_s(M_Z)$ above the present world average, one should recall that there are important uncertainties in this minimal supersymmetric GUT analysis. For one thing, there are in general important GUT threshold effects, which have been evaluated as

$$\delta_{\rm heavy} = \frac{3}{10\pi} \quad \alpha_{\rm GUT} \quad \ln\left(\frac{M_{H_3}}{M_{\rm GUT}}\right) + (\text{positive terms}) \tag{24}$$

in minimal supersymmetric $SU(5)$ [24,40], while $\delta_{\rm heavy}$ may be negative:

$$\delta_{\rm heavy} \simeq -4\% \tag{25}$$

in the $SU(5)$ missing doublet model of Eq. (18) [41] and in flipped $SU(5)\times U(1)$ [42]. As shown in Fig. 10, the missing-doublet model is in better agreement with the data on $\sin^2\theta_W$ and $\alpha_s(M_Z)$ than is the minimal $SU(5)$ model[43]. Moreover, there could easily be modifications

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Fig. 10. The missing-doublet model[30] provides GUT threshold corrections ϵ_g [41] that are in better agreement with the data [39] than is the minimal supersymmetric $SU(5)$ GUT[28].

of the unification conditions $\alpha_3 = \alpha_2 = \alpha_1$ due to non-renormalizable interactions scaled by inverse powers of m_P [44], which might yield an uncertainty

$$\Delta\alpha_s(M_Z) = \pm 0.006 \tag{26}$$

In view of all these uncertainties, I take the point of view that supersymmetric GUTs are still in very good shape, whereas it should be repeated that minimal non-supersymmetric GUTs are unquestionably in disagreement with the measured values of $\sin^2\theta_W$ and $\alpha_s(M_Z)$.

4. Baryon decay

As is well known, in minimal non-supersymmetric $SU(5)$ the preferred nucleon decay modes are:

$$\begin{aligned} &p \to e^+\pi^0 \ , \quad e^+\omega \ , \quad \bar{\nu}\pi^+ \ , \quad \mu^+K^0 \ , \ldots \\ &n \to e^+\pi \ , \quad e^+\rho^- \ , \quad \bar{\nu}\pi^0 \ , \ldots \end{aligned} \tag{27}$$

and the best available numerical estimate of the proton lifetime is[45]

$$\tau(p \to e^+\pi^0) \simeq (1.4 \pm 0.3) \times 10^{32\pm 1} \times \left(\frac{M_{\rm GUT}}{6 \times 10^{14}\ {\rm MeV}}\right)^4 \tag{28}$$

which is to be compared with the present experimental limit[46]

$$\tau(p \to e^+\pi^0) > 5.5 \times 10^{32} y \tag{29}$$

In view of the trend for higher-energy measurements[47] to find larger values of $\Lambda_{\overline{MS}}^{N_f=4}$, which could be as large as 400 MeV corresponding to $m_{\rm GUT} \simeq$ (4 to 8) $\times 10^{14}$ GeV, I no longer consider the conflict between Eqs. (28) and (29) to be conclusive. However, minimal non-supersymmetric GUTs are nevertheless excluded by the $\sin^2\theta_W$ argument discussed above.

The Grand Unification scale $m_{\rm GUT}$ is increased to about 10^{16} GeV in minimal supersymmetric $SU(5)$, yielding a lifetime for proton decay into $e^+\pi^0$ far beyond the present experimental limit. However, dimension-five operators in this model yield the alternative decays

$p \to \bar{\nu} K^+, \quad n \to \bar{\nu} K^0$ [48], for which the present experimental limits are less stringent than Eq. (29) [46]

$$\tau(p, n \to \bar{\nu} K) \gtrsim 10^{32} y \tag{30}$$

The limit (30) is (barely) compatible with theory for $M_X \lesssim 10^{16}$ GeV [49]. Missing-partner models[29,30] including flipped $SU(5) \times U(1)$ [31] greatly suppress dimension-five operators, which are no longer a problem. In the specific case of flipped $SU(5) \times U(1)$, the grand unification scale may be somewhat below 10^{16} GeV [42], particularly if one takes the lower end of the presently-allowed range of $\alpha_s(M_Z)$, in which case $p \to e^+ \pi^0$ and related decays may occur at observable rates, though with branching ratios different from minimal non-supersymmetric $SU(5)$ [50]. Therefore, the Superkamiokande detector about to start next year may finally be able to reassure us that protons are not forever!

5. Neutrino masses and oscillations

There is no good reason why neutrino masses should vanish, and grand unified theorists certainly expect them to be non-zero. The simplest form of neutrino mass matrix is the see-saw[51]

$$(\nu_L \ , \quad \bar{\nu}_R) \begin{pmatrix} m^M & m^D \\ m^D & M^M \end{pmatrix} \begin{pmatrix} \nu_L \\ \bar{\nu}_R \end{pmatrix} \tag{31}$$

where ν_R is a singlet right-handed neutrino field, and

$$m^D = g_{H\bar{\nu}\nu} < 0 | H_{\Delta I = 1/2} | 0 > \tag{32}$$

is a generic Dirac mass which is of order the charge-2/3 quark mass $m_{2/3}$ in many models, and m^M, M^M are $\Delta I = 1, 0$ Majorana masses which are expected to be of order $M_W^2 / M_X, M_X$, respectively. When one diagonalizes the matrix (31), one finds mass eigenstates of the generic form

$$\begin{aligned} \nu_L + 0 \left(\frac{m_W}{m_X} \right) \bar{\nu}_R \quad &: \quad m = 0 \left(\frac{M_W^2}{M_X} \right) \\ \nu_R + 0 \left(\frac{m_W}{m_X} \right) \bar{\nu}_L \quad &: \quad M = 0 \Big(M_X \Big) \end{aligned} \tag{33}$$

where "M_X" should be understood as anywhere between m_P and $O(\alpha/\pi)^2 m_{\rm GUT}$, depending on the model. Generically, (31), (32) and (33) yield the guess that

$$m_{\nu_i} \sim \frac{m^2_{\frac{2}{3}i}}{M_{X_i}} \tag{34}$$

for the three generations $i = 1, 2, 3$ of light neutrinos.

There are of course many more complicated models of neutrino masses incorporating more fields and/or more couplings, but this simple see-saw model accommodates in a very natural way the apparent deficit of solar neutrinos[53], and correlates it with the astrophysical wish

for a hot Dark Matter particle[54]. In my view, it is becoming increasingly difficult to retain an astrophysical explanation for the solar neutrino deficit, particularly in view of the strengthening helioseismological constraints on the solar model, including its central temperature. As reviewed here by Winter[55], the most appealing interpretation of the solar neutrino deficit invokes matter-enhanced neutrino oscillations[56]:

$$\nu_e \to \nu_{\mu \text{ or } \tau} \; : \; \Delta m^2 \sim 10^{-5} \text{ ev}^2 \;, \quad \sin^2 2\theta \sim \begin{cases} \quad 10^{-2} \\ \text{or} \\ \quad 1 \end{cases} \tag{35}$$

Theoretical prejudice (34) and the small values of inter-generational mixing angles observed in the quark sector favour the scenario

$$m_{\nu_e} \ll m_{\nu_\mu} \sim 3 \times 10^{-3} \text{ eV} \ll m_{\nu_\tau} \;, \quad \sin^2 2\theta_{e\mu} \sim 10^{-2} \tag{36}$$

Scaling the inferred value of m_{ν_μ} by m_t^2/m_c^2 and allowing $M_2/M_3 \sim 1/10$ leads naturally to the guess that

$$m_{\nu_\tau} \sim 7 \text{ eV} \tag{37}$$

as favoured in mixed dark matter models of cosmological structure formation, and

$$\sin^2 2\theta_{\mu\nu} \sim 10^{-0(3)} \tag{38}$$

which may be accessible to the new generation of accelerator neutrino oscillation experiments, CHORUS and NOMAD at CERN, and E803 at Fermilab[54].

As reviewed here by Winter[55], there are other suggestions of mass and oscillation effects in atmospheric neutrinos[57] and the LSND experiment[58], but I prefer to wait and see whether these claims become confirmed.

6. Further Dynamical Ideas

6.1. Electroweak Symmetry Breaking

It has been suggested[59] that the breaking of electroweak symmetry may be driven by renormalization of the soft supersymmetry breaking parameters[6] introduced earlier. This renormalization may resolve the apparent conflict between the preference of the super-Higgs mechanism for generating $m_0^2 > 0$ with the requirement that $m_H^2 < 0$ for the electroweak Higgs mechanism. The dominant renormalization effects are those due to gauge couplings and the top Yukawa coupling, which have opposite signs. If one follows the renormalization down to sufficiently low scales Q, large top Yukawa coupling may drive $m_H^2(Q) < 0$, triggering $m_W \neq 0$ [59]. This occurs at a scale Q hierarchically smaller than the input scale, so that

$$\frac{m_W}{m_P} = \exp\left(\frac{-0(1)}{\alpha_t}\right) \quad ; \quad \alpha_t = \frac{\lambda_t^2}{4\pi}$$

$$m_t = \lambda_t < H > \tag{39}$$

Typical dynamical calculations[59] yield m_t in the range now found by experiment.

6.2. Supersymmetry Breaking

The above mechanism for electroweak symmetry breaking requires soft supersymmetry breaking to be put in *a priori*: It is also possible that the scale of supersymmetry breaking may be determined by quantum effects[60]. Consider, for example, a model with no potential at the tree level in some flat direction in the space of moduli[61], so that it is independent of the generic supersymmetry breaking scale $\tilde{m}$:

$$\frac{\partial V_{eff}}{\partial \tilde{m}} = 0 \tag{40}$$

One then calculates the quantum corrections to the potential, which include the following terms at the one-loop level:

$$\delta V_{eff} \ni (\sum_B - \sum_F)\Lambda^4, (\sum_B - \sum_F)M^2\Lambda^2, (\sum_B - \sum_F)m^4 \ln\frac{m^2}{\Lambda^2} \tag{41}$$

where Λ is a cut-off scale which we may identify with M_P. The first term is absent in any supersymmetric theory, since the numbers of bosons and fermions are equal. The second term may be absent in specific supergravity or superstring models[62,63]. Assuming that this is the case[64], the effective potential enables the supersymmetry breaking scale and hence M_W to be determined dynamically[65].

Another suggestion is that supersymmetry breaking may occur non-perturbatively in a hidden sector of the theory, triggered by gaugino condensation[66]. It is even possible to imagine mechanisms which combine features of both of these scenarios. There are also ideas that, even within a fixed overall scale of supersymmetry breaking, the ratios of supersymmetry breaking parameters, i.e., the internal direction in super-flavour space, may be determined dynamically by radiative corrections[67].

6.3. Quark and Lepton Masses

The next step in a programme of determining dynamically all light mass scales is to tackle the fermion mass problem. For example, in many superstring models, the top mass is given by

$$m_t = \lambda_t < H_2 > \quad : \; \lambda_t = g \times f \text{ (moduli)} \tag{42}$$

where g is the gauge coupling and the moduli (vacuum parameters) may include radii of compactification and other quantities which are to be treated as quantum fields. These moduli are also often undetermined at the tree level. Perhaps these are also determined by quantum corrections, in much the same way as m_W and $\tilde{m}$ [68]. Such a scenario can be developed not only

for determining m_t, but also m_b and m_τ [69].

7. The Constrained MSSM

It is apparent from the preceding discussion that the MSSM contains many parameters beyond those already present in the SM: $m_{0_i}, M_{1/2_a}, \mu, \tan\beta, A_{ijk}, B_{ij}, \ldots$. In an attempt to reduce the dimensionality of this parameter space, it is desirable to impose necessary (plausible) phenomenological and theoretical constraints, which may include the following:

- No sparticles seen: We know from LEP1 that[46]

$$m_{\tilde{l}} \ , \quad m_{\chi^\pm} \gtrsim 45 \text{ GeV} \tag{43}$$

  and from the Fermilab $p\bar{p}$ collider that[70]

$$m_{\tilde{q}} \ , \quad m_{\tilde{g}} \gtrsim 150 \text{ GeV} \tag{44}$$

- No Higgs bosons seen: We know from LEP that[46]

$$m_{h,A} \gtrsim 50 \text{ GeV} \tag{45}$$

- Small FCNC: As mentioned earlier, this occurs naturally if the m_{0_i} are universal [7], but this assumption is not necessary[10].

- $b \to s\gamma$: The fact that this decay has been seen at a rate close to that predicted in the SM constrains MSSM parameters[71]. If there are no light sparticles, this constraint places a stringent lower bound on $m_{H^\pm}$, which may, however, be relaxed if some other sparticles are light.

- $\mu \to e\gamma$: This is not such a stringent constraint at the present time, but might become so in the future[72].

- $g_\mu - 2$: The forthcoming BNL experiment[73] should impose significant constraints on the sparticle spectrum[74] when it achieves its designed sensitivity.

- Neutron electric dipole moment: This imposes important constraints on possible CP-violating phase parameters in the MSSM[75], which depend on the overall sparticle mass scale.

- Cold Dark Matter density: The lightest supersymmetric particle is a good candidate for Cold Dark Matter, since R-parity guarantees its stability in many models, and its relic density lies in the desired range

$$0.1 \gtrsim \Omega_\chi h^2 \gtrsim 1 \tag{46}$$

  for generic values of the parameters[76]. The resulting constraints on the MSSM are quite sensitive to the magnitude of CP violation[77].

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Fig. 11. Fine-tuning upper limits on the possible sparticle spectrum assuming universal (dashed, solid) or non-universal (dash-dotted) squark masses[10].

One may add to the above phenomenological constraints some theoretical constraints, which are more speculative and hence more interesting. These include dynamical electroweak symmetry breaking[59], possibly supplemented by some no-fine-tuning requirement[78]:

$$\frac{\Delta M_W}{M_W} \lesssim \eta_i \ \frac{\Delta \ I_i}{I_i} \tag{47}$$

where I_i is some generic input parameter, and η_i parametrizes the amount of fine tuning. The absence of fine tuning was the basic phenomenological motivation for supersymmetry introduced in Section 1, but it is a matter of taste how to quantify it: should η_i be less than 1? 10? 100? 10^5? This argument certainly favours $m_0, M_{1/2} \lesssim$ a few hundred GeV [79], as seen in Fig. 11.

One might also postulate the dynamical determination of other scales, such as $\tilde{m}, m_t, \ldots$ as discussed above, or constraints arising from an infrared fixed-point analysis[81]. One may also impose some string-motivated Ansatz for supersymmetry breaking, such as

$$m_{1/2_a} = A = B = m_{3/2} \ , \quad m_{0_i} = 0 \tag{48}$$

at the string input scale[81]. This type of game is very exciting and predictive, but one should always remember that

$$\text{Prob (Result)} = \prod_{i=1}^{\infty} \ \text{Prob (Assumption)}_i \tag{49}$$

After expressing these words of caution, let us now look at some examples of constrained MSSM calculations.

Figure 12 shows an example[82] in which the measured value of m_t favours two possible solutions, one with small $\tan\beta$ and one with large $\tan\beta$. The large $\tan\beta$ solution has the additional attractive feature that it can accommodate equality between the t and b Yukawa couplings, as favoured in some string models. The two solutions yield different preferred ranges of sparticle masses, as seen in Fig. 12. Another example of a MSSM scenario[83] is shown in Fig. 13, where the possible masses of the sparticle species are plotted as a function of the lightest chargino mass. In this scenario, the right-handed sleptons have only barely escaped detection at LEP1 and the lightest chargino should be discovered at LEP2, as should the lightest

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Fig. 12. Results from a constrained MSSM[82], indicating two preferred regions at small and large $\tan\beta$, the latter being consistent with equal t- and b-quark Yukawa couplings.

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Fig. 13. Results for the sparticle spectrum in a constrained MSSM[83].

supersymmetric Higgs boson. This scenario also suggests that the $p\bar{p}$ collider at Fermilab may be able to see dilepton and trilepton events due to sparticle pair production and decay.

8. String Theory

This is the only candidate we have for a Theory of Everything (TOE). It is an apparently consistent quantum theory of gravity, at least at the perturbative level and possibly also non-perturbatively. It provides a framework for tackling the thorny issues of space-time foam, cosmology, the cosmological constant, etc. It also provides a framework for unifying the particle interactions. However, whereas initially it was thought that there might be a unique string model, namely the $D = 10$ E_8 heterotic string[84], or perhaps only a few models, subsequently many consistent string models have been found. These include a multitude of apparently consistent compactifications of the original heterotic string[85], but the most general formulation of such models is as heterotic strings directly in four dimensions[86]. These different models may be regarded as different vacua, i.e., solutions of the classical equations for the moduli, of the same underlying string theory. All couplings correspond to expectation values of fields (moduli), for example for the gauge couplings g_i:

$$g_i^2 = \frac{k_i^2}{< S >} \tag{50}$$

where the k_i are Kac-Moody level parameters to which we return later, and S is a type of

dilaton field.

In all this confusing thicket of string models, one can make some generic predictions. For example, the string unification scale at which $\alpha_i = \alpha_j = \alpha_{\text{graviton}}$ can be predicted[87]

$$m_{SU} \simeq 5 \times 10^{17} g_{\text{GUT}} \;\; \text{GeV} \tag{51}$$

There is also a generic prediction for m_t, as mentioned earlier

$$\lambda_t = g_{\text{GUT}} \times f \;(\text{moduli}) \tag{52}$$

which leads to the qualitative expectation that $m_t/M_W = O(1)$, with the possibility of dynamical determination discussed earlier.

Among the techniques used in string model building are the compactifications of the $D = 10$ heterotic string[85] mentioned earlier, orbifolds[88], free fermions on the world sheet[89], etc., all of which have been used to produce models with gauge groups of the form $SU(3) \times SU(2) \times U(1)^n$ [90]. Making a string GUT is more problematic, because these typically require adjoint Higgs representations (e.g., the **24** of $SU(5)$), which are not available if we maintain space-time supersymmetry and restrict ourselves to the level $k_i = 1$ [34]. This was a motivation for resuscitating flipped $SU(5) \times U(1)$ [31], which, as discussed earlier, also has an elegant missing-partner mechanism, a see-saw neutrino mass matrix, and proton decay at a rate which may be accessible to Superkamiokande if $\alpha_s(M_Z)$ is in the lower half of the range presently allowed by experiment. However, this and other string models lose (or at least weaken) the minimal supersymmetric GUT prediction for $\sin^2\theta_W$. For this and other reasons, theorists have been trying to construct supersymmetric $SU(5)$ and $SO(10)$ GUTs using higher-level Kac-Moody algebras[91]. The models found so far either have more than three generations[92] or other additional chiral stuff[93], but developments in this quest are very promising and should be watched.

Let us turn finally to a dramatic new development in string theory, which may diminish significantly the apparent proliferation of string models. As discussed here by Rubakov[94], it has recently been realized that gauge theories with extended supersymmetries have[95] amazing duality properties[96], which interrelate strong- and weak-coupling descriptions of the same physics. It has also been realized that string theories possess many such duality properties[97]. These include so-called T duality, of which the simplest example is the equivalence between a string compactified on a loop of radius R and one compactified on a loop of radius $1/R$. This symmetry relating different moduli is believed to be elevated to a symmetry at least as large as $SL(2, Z)$. String theory may also possess an S duality interrelating strong and weak coupling $< S > \leftrightarrow 1/ < S >$ in Eq. (50), which may also be elevated to $SL(2, Z)$ [98]. Even more excitingly, many examples have been found of string-string dualities, namely equivalences between different types of string, one weakly coupled and one strongly coupled. Figure 14 is a provisional map of some string dualities, which apparently include, for example, an equivalence between the $D = 10$ heterotic string compactified on a four-dimensional torus T_4 and the type IIA string compactified on a K_3 manifold[99], as well as many others. One of the most striking dualities is that between the heterotic $SO(32)$ string and the type I $SO(32)$ string[100], with spinors of the former interpreted as solitons of the latter, and the type IIB string appears

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Fig. 14. A provisional map of some of the string dualities recently discovered.

to be self-dual[101]. There are also duality symmetries[100] relating string theories with $D = 11$ supergravity[102] and with supermembrane theories! [103]

This is a rapidly-moving field with many new results being obtained[97]. It offers the possibility that many different types of string model may simply be re-expressions of the same underlying theory, whose most basic formulation may well lie beyond the concept of string. Any such development could only comfort the belief that we have found the TOE.

9. Conclusions

There are good theoretical and experimental motivations to hope that we are finally on the brink of discovering new physics beyond the Standard Model. Precision data from LEP1 and elsewhere suggest that the Higgs boson is light, in agreement with the prediction of supersymmetry, and may well be accessible to LEP2. The consistency between measurements of $\alpha_s(M_Z)$ and $\sin^2\theta_W$ and the predictions of supersymmetric GUTs is certainly encouraging, even if it does not yet enable us to determine with any accuracy the scale of supersymmetry breaking. The persistent solar neutrino deficit seems ever more difficult to explain using astrophysics, and may be the harbinger of neutrino masses and oscillations.

The exploration of large new domains of supersymmetry and GUT parameter space is about to start, with the advent of LEP2, a new generation of accelerator neutrino oscillation experiments pioneered by CHORUS and NOMAD, and a new generation of large underground experiments pioneered by Superkamiokande and SNO. Will our luck finally change? Will the next meeting in this series become the first Slepton-Photino Symposium?

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